

# “Adaptive time-space algorithms for the simulations of multi-scale reaction waves”

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## **Abstract:**

Numerical simulations of multi-scale phenomena are commonly used for modeling purposes in many applications such as combustion or plasmas physics. These models raise several difficulties created by the high number of unknowns, the wide range of temporal scales due to detailed chemical kinetic mechanisms, as well as steep spatial gradients associated with very localized fronts of high chemical activity. Furthermore, a natural stumbling block to perform 3D simulations with all scales resolution is either the unreasonably small time step due to stability requirements or the unreasonable memory requirements for implicit methods. In this work, we introduce a new resolution strategy for multi-scale reaction waves based mainly on time operator splitting and space adaptive

multiresolution. It considers high order time integration methods for reaction, diffusion and convection problems, in order to build a time operator splitting scheme that exploits efficiently the special features of each problem. Based on theoretical studies of numerical analysis, such a strategy leads to a splitting time step which is not restricted neither by fast scales in the source term nor by restrictive stability limits of diffusive or convective steps, but only by the physics of the phenomenon. Moreover, this splitting time step is dynamically adapted taking into account a posteriori error estimates, carefully computed by a second embedded and economic splitting method. The main goal is then to perform computationally efficient as well as accurate in time and space simulations of the complete dynamics of multi-scale phenomena, considering large simulation domains with conventional computing resources and splitting time steps dictated by the physics of the phenomenon and not by stability constraints associated with mesh size or source time scales. Applications will be presented in the fields of combustion waves and plasma discharges dynamics and the question of high-performance computing using such numerical methods will be discussed and illustrated.

## **Biosketch:**

Marc Massot obtained his PhD in Applied Mathematics from Ecole Polytechnique, France, in 1996. After a year at Yale University, Department of Mechanical Engineering, he obtained a CNRS position in the Applied Mathematics Laboratory of the University of Lyon, France, where he stayed until 2005. He was offered an Associate Professor position at Ecole Centrale Paris when he installed a mathematics team in the EM2C mechanical engineering laboratory. He was Visiting Professor at the Center for Turbulence Research, Stanford University, in 2011-2012 and created and chaired the Fédération de Mathématiques de l'Ecole Centrale Paris between 2013 and 2016. He initiated the Computing Center (Mésocentre) of Ecole Centrale Paris in 2010, of which he was the deputy director until 2016 and he is scientific adviser at ONERA DEFA and scientific collaborator at Maison de la Simulation since 2013. Full Professor since 2011, he has been recruited on a full Professor position at Ecole Polytechnique, Centre de Mathématiques Appliquées in 2017. His main fields of research are mathematical modeling and numerical analysis, analysis of PDEs and dynamical systems for multi-scale systems, scientific computing and high performance computing with applications in combustion, two-phase flows, plasma physics and biomedical engineering.

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